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ESP OVER DISTANCE: A SURVEY OF EXPERIMENTS PUBLISHED IN ENGLISH

BY

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and

APPENDIX TO DR. OSIS' ARTICLE: A STATISTICAL MODEL FOR EXAMINING THE RELATION BETWEEN ESP AND DISTANCE

BY

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ESP Over Distance: A Survey of Experiments Published in English

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ABSTRACT: ESP experiments published in English and involving distances of at least a hundred yards were surveyed. The purpose was to establish the available empirical trends and to clear the ground of misconceptions about ESP over distance. The bulk of the data derived from experiments using five-alternative-choice methods, such as guessing ESP cards. These scores were grouped into distance classes and evaluated by new statistical methods devised by Dr. Malcolm E. Turner, Jr., Professor of Statistics at Emory University. Turner's model (see Appendix) is a method for describing any lawful relationship between ESP and distance. Turner found that the scores in the data surveyed decline significantly with distance. The decline does not follow the customary inverse square law of the known radiant energies, but rather an "inverse two-fifth's law"; i.e., the scores decline by somewhat less than the inverse square root. However, the inverse two-fifth's law cannot be extended outside the surveyed sample to ESP in general because of severe limitations in the data available.

In cases where the same subject performed ESP tests over two different distances within one experiment, in all instances the scores at the longer distances were lower than those at the nearer distances. Experiments using other than five-alternative-choice techniques generally showed the same trend of decline with distance. A challenging need for new creative thinking and better experimenting with ESP over distance is emphasized.—Ed.

THE PROBLEM AND ITS PERSPECTIVE

One of the most fascinating characteristics of ESP is that it can overcome long distances in space, and seemingly in time also, penetrating the future. A puzzling aspect of man's personality is that by just closing his eyes he may on occasion envision correctly what is happening on the other side of the world or what will occur in the future. The available evidence from experiments and spontaneous cases seems to justify the assumption that ESP can reach around the globe (5, 21, 26, 31, 32, 38).

This apparent sovereignty of ESP over space and time has, over the centuries, spurred the creative imagination of researchers and philosophers (9, 13, 20, 37, 54, 55). Many able individuals have rightly considered the spatial and temporal properties of ESP to be among the most important keys to the riddle of the nature of human personality. For them, the "reach of the mind," with its virtual omniscience of far lands and the distant past and future, gives

I am very grateful to Dr. Malcolm E. Turner, Jr., of Emory University, who challenged my conventional ideas about ESP over distances and so instigated this survey. His help as consultant and collaborator in the evaluations is highly valued. The late Dr. E. B. Foster, Mrs. M. J. Mason, Mrs. J. E. Nteger, and Mr. J. M. Smith also gave valuable assistance which is deeply appreciated.

ESP Over Distance: Survey of Experiments

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back to man some of the dignity and grandeur lost in modern scientific concepts of personality. I believe that this line of thought is sound and constructive, but only up to a point.

The space-time problem has proved to be one of the most dangerous pitfalls in parapsychology because it has tempted researchers to speculate far beyond what our factual knowledge of ESP warrants. We know for certain only a few facts about ESP as related to space and time, and even these facts are entangled with a multitude of other variables inseparably interwoven in the data. Let us beware of constructing gigantic castles with just these few solid bricks of knowledge, added to a whole warehouse of building materials from philosophies of the past. There is no reason why we shouldn't speculate if we admit that this is what we are doing; however, it becomes dangerously confusing if strong empirical evidence which simply does not exist is claimed as a basis for these lofty castles.

I am fully aware of the controversial nature of this topic and, in fairness to the reader, I want to state my own bias. When, as a young student, I wrote my dissertation on the hypothesis of ESP, I believed that the independence of ESP from distance in space and time was an established fact. I took it as the cornerstone of my thesis. After coming to the United States, the original sources became available to me; I checked there the validity of claims for clear-cut empirical evidence for ESP's independence of space. To my horror I found none.

The power of our cultural heritage, or *Zeitgeist*, to distort thinking in parapsychology is clear in the case of the distance problem. I believe, therefore, that we definitely need clarification of the available empirical facts about ESP over distances; this is the purpose of the present survey. In order to escape the spell of the *Zeitgeist*, I went straight to the *primary data* in the published experiments sampled and by-passed the *opinions* of the experimenters. This was necessary in order to clear the ground for new creative theoretical thinking and better future experimentation.

DELINEATION OF THE PROBLEM

The ESP communication system clearly has three parts. At one end is the internal functioning of the percipient, who acquires information by ESP. At the other end is the stimulus, for example, an ESP card, an accident, or an idea in a friend's mind. The system's third part, to which this paper is devoted, is the unknown factor which brings both ends of the communication system together. In my opinion no one yet knows what this factor is. Among other possibilities, it might be visualized as analogous to the light

rays which connect our eye with the object seen, or to the sound waves which connect the tones of a violin and the listener. Modern communication theories are accustomed to calling this mediating factor a "channel." I think we can adopt the term for ESP also. Let us bear in mind that this channel does not have any spatial properties, such as those of a pipe or canal. Radio waves are called a channel in spite of their being distributed around the globe. This paper is concerned only with the channel in the ESP communication system, whatever it might turn out to be, and not with the stimuli or the subjective, internal factors. Further, as I have already said, I will survey and evaluate *only* the experimental data pertinent to the channel factor, and *not* the hypotheses and speculations which have been based upon these data. Theoretical discussions are very necessary, of course, but I will wait to develop some of my own in another paper.

PROCEDURE

For relevant empirical material on distance in ESP the parapsychological literature in English was surveyed: this included research journals and books. The literature in other languages was touched upon only lightly because of linguistic limitations.

In ESP research two basic types of stimuli are used: (a) alternative-choice, e.g., ESP cards, in which the subject knows the alternative stimuli, but must ascertain the order of presentation; and (b) free stimulus material, e.g., drawings, in which the subject does not know the stimulus population and must indicate the nature of the stimuli. These two types of research are reviewed separately, with primary emphasis on alternative-choice methods and visual stimuli. Experiments with ESP cards (or other targets with one-fifth probability for guessing correctly) form the backbone of the survey, and special attention is given to research in which the same subject did ESP tests involving different distances within the same experiment. A few distance experiments were found where playing cards, numbers, letters, and clock cards were used, but these will be mentioned only rather briefly since there were too few instances to permit grouping in distance classes. Some of the distance experiments using free stimulus material are reviewed, but with emphasis only on those in which quantitative evaluation was carried out.

Excluded from the survey are all experiments not designed for distance comparisons and in which little-known response methods (ESP in dreams, map dowsing, hypnosis induced by ESP, etc.) were used. Mediumistic and so-called psychometry experiments were also excluded since their results are open to other interpre-

tations than ESP over distances. Broadcast experiments were sampled but not included in the tabulations since none in which distances were evaluated were found to have been performed under acceptable conditions. Experiments in which distance ESP was coupled with precognition or psychokinesis were not deemed suitable for inclusion because of the obvious confounding of variables. Finally, no tabulation of spontaneous ESP experiences was attempted; this would be a herculean task in itself and would in any case involve great difficulties of interpretation in terms of the distance factor.

Having decided on the type of research material to be included in the survey, another problem arose. What is the minimum distance between subject and target in an experiment which will warrant its being called a "distance experiment"? Subjects usually experience little change in attitude if they are separated from the targets by only a few yards; but they seem to consider it a "real distance" if the cards are placed in another building. Therefore, in order to minimize the possible effect of the attitude factor, a hundred yards—the distance assumed to separate two buildings—was chosen as an arbitrary lower limit.

The distance within the first mile seems to be rather interesting as a factor in the transmission of known energies. Therefore, the distance from a hundred yards to one mile was used for the first "distance class." The range of the other classes is a thousand miles. If the original source reported the data within a distance range of, say, 500-1400 miles, the middle of the range—in this case 950 miles—was taken as the basis for classification in the present survey.

Distances were measured as around the globe; that is, on the circumference. Of course, the distance through the globe, the diameter, is somewhat smaller, but as the known energies used for communication, e.g., radio waves, bounce around the globe, it was decided to use this same distance measure for ESP in order to permit direct comparison.

The minimum length of experiments accepted for evaluation in the survey is twenty runs (500 guesses). However, this rule was liberalized for some experiments which are mentioned, but not included in the main evaluations.

No special measures were taken for selection in terms of the quality of the experiments as I wanted to include *all the data from which the claims of no lawful relationship between ESP success and distance had originated.*

In a sense this survey will give rather inaccurate information because of limitations in the data. Approximately half of the tabulated material involves multiple calling for the same set of targets,

and this could not be taken into account in our evaluations. Experiments 1 and 2 (in Table 1 below) are insufficiently described in the original publications. Some experiments involved highly selected subjects and others unselected groups. Conditions varied widely. However, we simply do not have anything better to stand on in evaluating claims of evidence for the independence of ESP from distance.

RESULTS

Card Experiments with Five Alternative Choices

Experiments in which cards with five alternative choices were used as stimuli are summarized in Table 1. The scoring levels according to the distance classes are distributed in definite non-

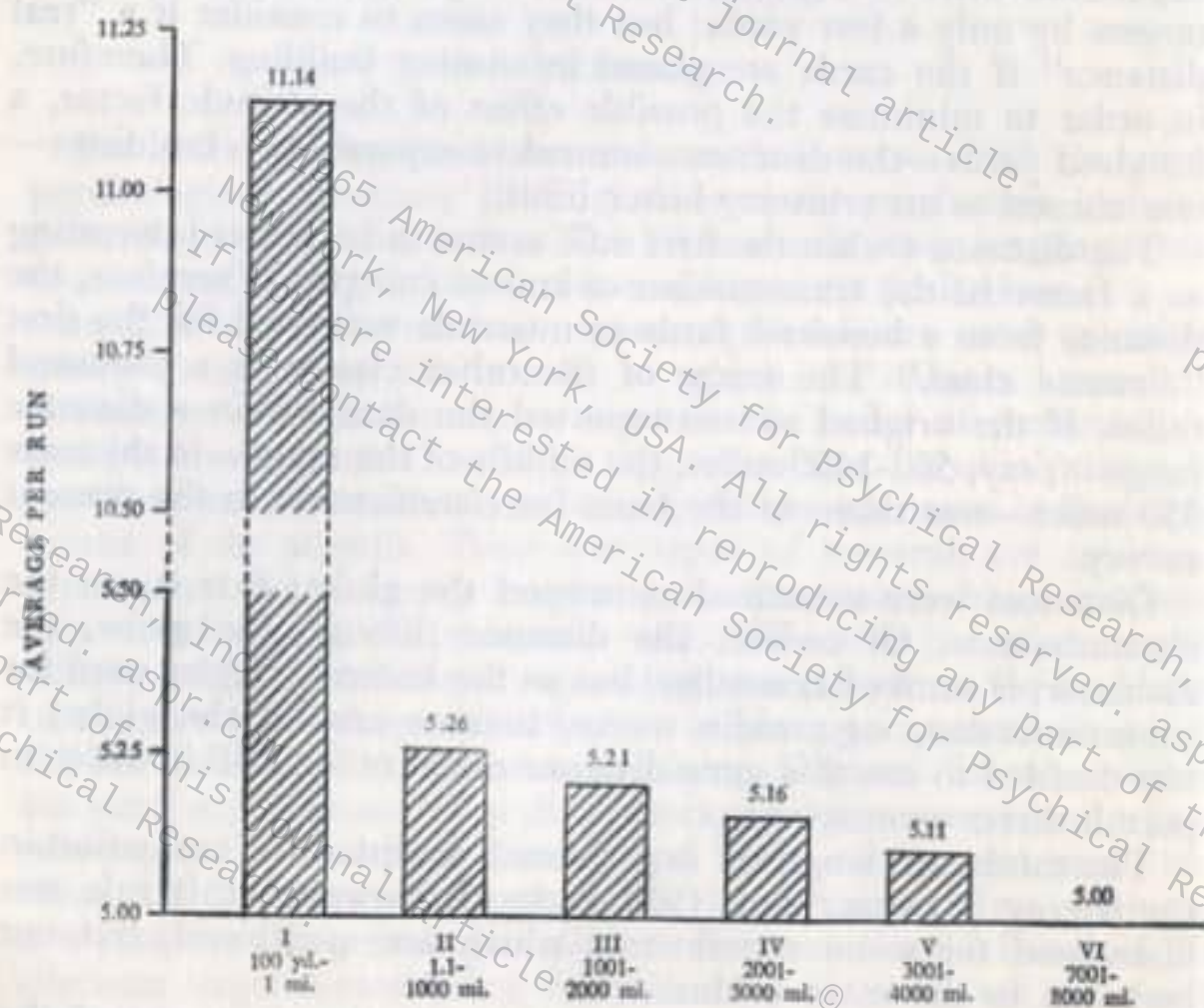


Fig. 1. ESP scores according to distance classes (only experiments with five alternative choices are included).

chance fashion. When the distance increases, the class average decreases consistently from class to class without exception (Figure 1). Most decline is registered between the first two classes; that is, between the 100-yards-to-1-mile class and the 1.1-mile-to-1000-

Table 1
ESP SCORES ACCORDING TO DISTANCE CLASSES*

Experiment No.	Experiment	I 100 yd.-1 mi.		II 1.1-1000 mi.		III 1001-2000 mi.		IV 2001-3000 mi.		V 3001-4000 mi.		VI 7001-8000 mi.	
		Runs	Avg.	Runs	Avg.	Runs	Avg.	Runs	Avg.	Runs	Avg.	Runs	Avg.
1	Rhine: Extra-Sensory Perception (35)	74	7.54										
2	Rhine: Tarkio Series (36)	74	18.24	34	6.38								
3	Rhine: Duke Series (36)	10	5.30	1045	5.52	28	4.93	527	5.16	353	5.07	24	6.17
4	Rhine, Pratt: Pearce-Pratt Series (39)	40	6.15	402	5.22	190	5.25			1000	5.09	24	3.83
5	Gibson (16)												
6	Ries: Series A (40)												
7	Soal, Bateman: Telephone Series (50)			64	4.66								
8	Cambridge Series (50)			48	7.19								
9	Antwerp Series (50)												
10	Rhine, Humphrey: Marchesi Series (38)			339	4.77								
11	Duke Subjects (38)			299	4.72								
12	McMahan, Rhine: Marchesi Exp. (26)			301	5.16								
13	McMahan, Bates: Marchesi Exp. (24)			854	5.25								
	Skibinsky: Names (48)												
	ESP Symbols (48)												
	Kahn: Harvard Exp. (18)												
	Osis: Haackel Exp. (31)												
	Osis, Pienaar: Slow Rate (32)												
	Rapid Rate (32)												
	Totals	198	11.14	3386.56	5.26	218	5.21	527	5.16	1697	5.11	48	5.00

* Only experiments with five alternative choices are included.

mile class. Actually, the first of these classes ranges only from one hundred yards to a quarter of a mile, as there are no data for the rest of the range.

Dr. Malcolm E. Turner, Jr., Professor of Statistics at Emory University, has developed a statistical model for evaluation of the problem of ESP over spatial distances (see Appendix). He is the sole author of this model. The basic idea of the model is to express in mathematical terms any lawful relationship that ESP might have to distance. The model utilizes modern information theory, as well as the latest advances in statistics.

The gist of the procedure in over-simplified, non-mathematical terms is this: ESP scores are transformed by means of appropriate statistical operations into amounts of information transmitted, as expressed in "natural units" or "nits." This is necessary in order to provide a basis for calculating statistics which describe important trends in the relationship of ESP to distance. One of these statistics is π , which Turner defines in the Appendix: " π is a constant we may term the 'extrasensory perceptivity' and is a measure of the 'power' of the object-subject system in a particular situation." It is necessary to calculate this "extrasensory perceptivity" in order to describe, by another statistic, δ , precisely the way in which ESP declines over distances, if indeed it does.

This statistic δ , or "proportionality constant," is Turner's basic tool for pinning down mathematically any distance law. With it one can sweep across all the major possibilities of a distance law to find precisely that one which fits the data best. For example, if δ is found to be 0, ESP is not related to distance; if δ is equal to -2, the result is the inverse square law characteristic of known energies when radiating in all directions. Other values of δ would show, for example, whether ESP decreases over the distance rapidly or slowly; uniformly over any distance; with a rapid decline at first, leveling off to a slight decline, etc.

Dr. Turner kindly volunteered to evaluate the survey data represented in Table 1. The evaluations were done in accordance with his model, and the steps are outlined in the Appendix. He found that the following trends occurred:

1. ESP scores are significantly related to distance.
2. "Extrasensory perceptivity" or "power constant" $\pi = 0.608$.
3. "Proportionality constant," which "indexes the class of distance laws," $\delta = -0.398$.
4. Some chance variation (error variance) could, of course, affect the magnitude of δ . Its true value most likely is within the usual statistical confidence limits. At the 95% confidence limit, the true δ should be somewhere between the following values: $P(-.409 \leq \delta \leq -.387) \cong .95$.

At the 99% confidence limit, which is more conservative, the finding is: $P(-.416 \leq \delta \leq -.380) \cong .99$.

5. Turner calculated on the basis of these statistics the way in which the average run score would vary with distance. The predicted mean run score as a function of distance x , is: $\bar{m}_x = 5e^{0.608x - 4.397}$, where e is the base of the natural logarithm.

Turner concludes that $\delta = -.4 = -2/5$ seems to be as good a choice as any for describing the ESP distance law; in other words, ESP obeys the inverse two-fifth's law in the data of Table 1.

Let us sum up: In the data evaluated, there is a significant relationship between ESP scores and the distance between subject and stimulus. The relationship indicated is not of the usual inverse square type, but more like that of an inverse two-fifth's. In other words, in these experiments the ESP scores decreased somewhat less than by the inverse square root of the distance.

The crucial question arises: Can we generalize from the results of this survey sample that ESP effects are inversely related to distance? The limitations and weaknesses of the published data have been pointed out above. In my opinion the data are too scant and defective to warrant safe conclusions as to ESP in general.

These same data have been the basis for the *Zeitgeist* interpretation that ESP is independent of distances. Obviously Turner's analysis does not support the independency hypothesis at all; if anything, what is indicated is the dependency of ESP on distance. However, Turner's evaluations set a challenging example of what can be extracted from the data of better-designed future experiments, aimed at the channel factor in ESP.

Comparison of Scores of the Same Subject over Two Different Distances (five alternative choices)

In the data presented in Table 1 variables associated with the subjects (individual differences, attitudes, etc.) are neither balanced nor statistically controlled. The first distance class is filled exclusively with the scores of some of the best subjects ever known in parapsychology. This factor alone might explain the high scores in this class.

If we could sufficiently balance factors other than distance influencing ESP scores, and at the same time vary only the distance between subjects and targets, much more dependable information would be obtained. I do not know of any published experiment where this requirement was fully met. The closest approximations to it are the cases in which the same subject has guessed cards at two different distances within one experiment.

There were seven such cases in the surveyed publications (see

Table 2
ESP SCORES OF THE SAME SUBJECT OVER TWO DIFFERENT DISTANCES⁺

Experiment	Near Distance			Long Distance			Average Differences++	CR _d ++
	Distance	Runs	Avg.++	Distance	Runs	Avg.++		
1	0	50	6.12	700-2000 mi.	72	5.58	.54	1.47
Gibson: Subject J (16)	0	25	5.80	700-2000 mi.	42	4.74	1.06	2.11*
Subject S (16)	0	24	6.46	700-2000 mi.	68	5.20	1.26	2.65**
2	100 yd.	30	8.77	250 yd.	44	6.70	2.06	4.36**
Rhine, Pratt: Pearce-Pratt Series (39)	0	64	6.23	70 mi.	64	4.66	1.58	4.47**
3	0	48	7.27	200 mi.	48	7.19	.08	.20
Soal, Bateman: Cambridge Series (50)	0	55	6.13	4000 mi.	100	5.24	.89	2.64**
Antwerp Series (50)								
4								
Osis: Haackel Exp. (31)								
Totals		296.4	6.60		438	5.52	1.08	7.16**

+ Only experiments with five alternative choices are included.

++ The average scores, average differences and CR_d's were rounded for sake of convenience in presentation; hence, they will not check exactly.

* Significant at the .05 level.

** Significant at the .01 level.

Table 2). In cases 1 and 2, and in the Antwerp Series of 3, near and far distances were not used in the same session. In the Cambridge Series of case 3, and in case 4, the long-distance calls were done in the favorable first half of the session and the near-distance calls afterwards. In case 1, the procedures are only sketchily described. In case 3 (Cambridge Series) some synchronization errors might have interfered with the distance part of the experiment.² Only in case 3 (Antwerp Series) did the same agent look at the targets at both distances.

This kind of comparison clearly indicates the same trend as in the previous analyses. In all cases the scores at the longer distance are lower than the scores for the near-by targets (Table 2 and Figure 2). In five out of seven cases the scoring differences are individually significant (in one case $P < .05$; in four cases $P < .01$). The CR of the difference of the total is 7.16. The consistency of the trend is obvious.

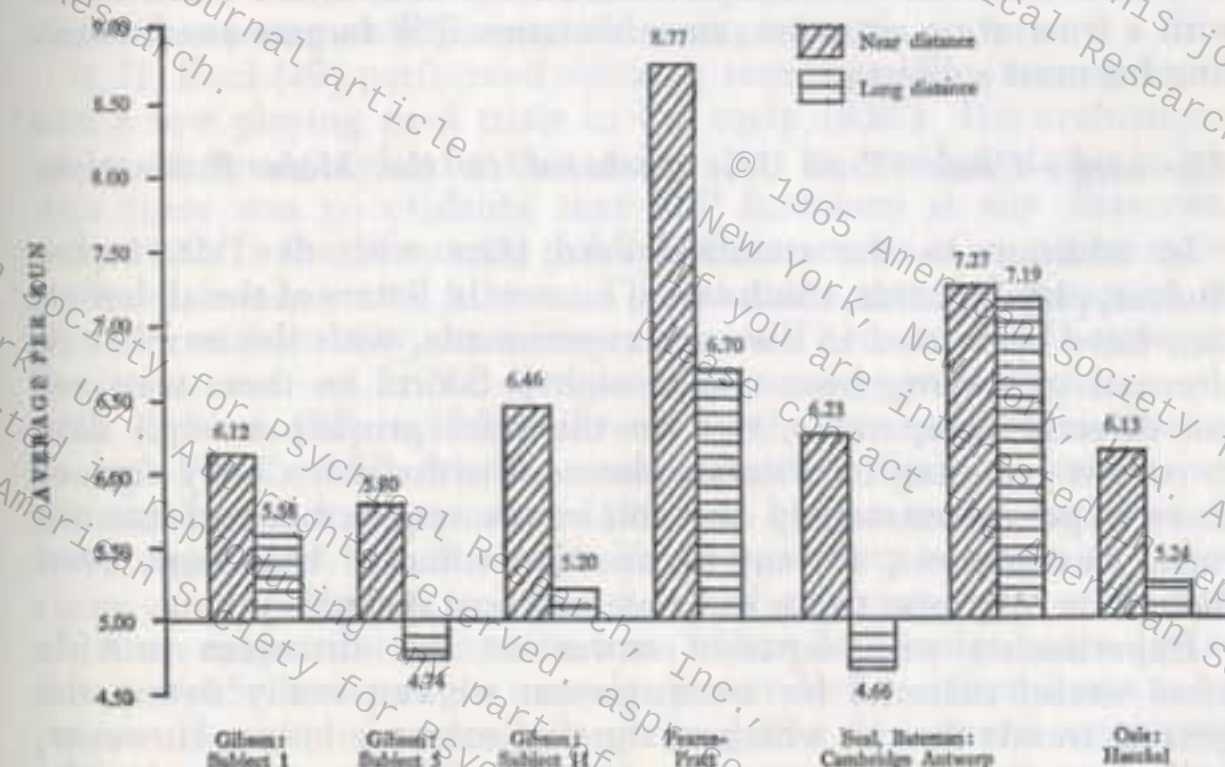


Fig. 2. ESP scores of the same subject over two different distances (only experiments with five alternative choices are included).

There is in Table 2 a considerably smaller number of variables that could account for the differences observed than there is in Table 1. Nevertheless, these differences still might be explained in terms of the subjective variables only.

The most serious counterhypothesis is raised by J. B. Rhine and

² If we exclude case 3 (Cambridge Series) from the analyses, the findings will not change.

others (36, 50, 59) to the effect that the subject's *attitude* toward the distance condition could in itself explain any scoring trends observed. This is a sound hypothesis, but it still lacks empirical validation. For example, if the subject feels challenged and alerted by the task of "overcoming distance" by ESP, he might on this account score best on distance targets. On the other hand, if he is adversely affected by the frustrating situation of distance ESP, the scores might go down. The importance of the attitude factor in distance telepathy was already recognized by E. M. Sidgwick and A. Johnson in 1892. At that time they called it "... the effect of the distance on the imagination of the agent or the percipient" (46, p. 547).

The obvious way out of this difficulty would be to set up experiments in which the variables of distance and attitude are controlled. We could ascertain the subject's attitude and take it into account in statistical assessment; and by the same token we could also find out what the subject habitually does when confronted with a frustrating situation, since distance ESP targets *are* frustrating for most subjects.

Alternative Choice Tests Not Included in the Main Evaluations

In addition to the standard card tests with five alternative choices, playing cards, clock cards, numerals, letters of the alphabet, etc., have been used in distance experiments, with the number of alternatives varying from two to eighty. Scores on these tests are not directly comparable, nor do the tests provide enough data to permit grouping in distance classes. Furthermore, very few of these experiments satisfy the minimum requirement of twenty runs. Nevertheless, I want to mention some of this work even though its relevance to the distance problem is slight.

Experiments with separate scores on two distances provide more useful material for comparisons: we can easily detect the general trend; that is, which of the two scores is lower. However, unified statistical treatment is not possible here because the probabilities of hitting are so heterogeneous and the reported statistics so varied. Therefore, only a simple tabulation of the higher and lower scores is noted.

In 1938 R. Shulman (45) performed ESP experiments using sound stimuli with five alternative choices over distances from a hundred to four hundred yards. These were compared with tests using screened color ESP cards, where distance was not involved. Unfortunately, there is very little known about ESP and auditory stimuli, as nearly all ESP experiments have utilized visual stimuli. We cannot include Shulman's data in the main evaluations because

stimulus and distance variables were confounded. However, his results follow the general trend of decline already observed in the survey: 345 runs, no distance, average score: 5.17; 223 runs, over distance, average score: 5.08.

Some experiments with ESP cards (25) and with other types of cards (14, 15, 29) obviously involved distances, but either the data were not separated into distance categories or the distances were not stated.

F. L. Usher and F. P. Burt (56) in 1907 performed a pioneering experiment with playing cards over distances ranging up to 960 miles. The number of trials was very small: 36 trials, in the same room, 9 complete hits; 60 trials, over distance, 4 complete hits. (This involved multiple calling of some cards.) The trend is obvious.

J. E. Coover (11) reported in an appendix to his larger work a brief experiment using playing cards with the face cards omitted ($P = 1/40$). The distances varied from twenty-seven to ninety miles. He obtained 5 hits in 149 trials. The scores on the various distances are not clearly separated in the report.

S. G. Soal (49) performed distance tests involving free material and a few playing card trials in the early 1930's. His evaluations are much too complex for this review, but he concluded that in his data there was no evidence that ESP functions at any distances.

In 1932 T. Besterman (2) published the results of an experiment between London and Athens. The stimulus material consisted of pictures with twenty alternative choices. There were 674 guesses: deviation was -4.7 . No distance comparisons were made.

C. B. Nash (30) in 1961 followed up the star subjects in Chesebrough-Pond's ESP television contest with distance tests using the numbers 0-9 as stimuli. In 180 runs he obtained a deviation of -41 , associated with $P = .04$. Further, twenty subjects at a distance ranging from 6 to 520 miles had a deviation of -16 , and nineteen subjects at a distance from 680 to 2400 miles had a deviation of -25 . A very slight non-significant decline effect seems to be present, but the numerically larger negative deviation is associated with the greater distance. Nash's test was meant to be a test of GESP; however, the instructions to the subjects were like those devised for precognition tests, emphasizing future comparison.

D. Michie and D. J. West (27) performed in 1957 a mass television ESP test, and followed up their star subjects. Clock cards were used as stimuli in the follow-up. One high-scoring subject, Mr. Downey, was extensively retested. His "mean divergence score deviation" over a distance of approximately a hundred miles was 0.8125; at no distance, 0.7627. The difference is extremely small and in the opposite direction from the general trend of this survey.

In 1963 A. E. H. Beksley (3) reported an experiment on long-

distance ESP during sleep. This involved the subject's awakening each night at a certain time randomly selected by the experimenter in Johannesburg, South Africa. During the first series the subject was traveling from South Africa to Holland; distance varied and were quite great, up to five thousand miles. During the main series the subject remained in South Africa at a distance of nearly a thousand miles. Bleksley's evaluations are too involved to describe in this survey, but more ESP success was reported at the constant, but shorter distance. Bleksley gives plausible psychological explanations for this difference: He met the subject only after the series at the longer distances was completed.

G. L. Mangan (21) performed a distance experiment over approximately 2150 miles. He used dual-aspect targets with 1/5 probability of hitting each aspect and 1/25 for hitting the whole card. The subject was instructed to make one guess a day for a hundred days. Ninety-five trials were scored and resulted in 13 complete hits, with 3.8 expected by chance. This result is associated with $P = .000002$. Partial hits, as well as displacements, were also evaluated. The number of trials is too small for inclusion in the main evaluations. However, the rate of ESP scoring over so long a distance is very high indeed and this small experiment stands out as very impressive evidence for long-distance ESP.

Experiments Using Free Stimulus Material

Distance experiments with free material are numerous, but difficult to evaluate. Most reports do not include quantitative evaluations. In cases where quantitative evaluations are given, subjective judgment on the part of the judges is always involved, leaving room for an alternative interpretation—the judge's own ESP exerted during the judging process. Of course, the judge's ESP influence is less likely to be a factor in experiments where complete similarity between response and stimulus is demanded, as in M. C. Marsh's main evaluations (22, 23).

Distance tests with free stimulus material were attempted as early as the 1880's by S.P.R. researchers, and F. Podmore's book (33), published in 1902, contains an entire chapter summarizing the results of many attempts to produce "telepathic effects at a distance." The Miles-Ramsden series (28) was a later long-range exploration of the same type. R. Warcollier's experiments (58) in the 1920's and 30's with free material made many useful contributions to the field of parapsychology, but their lack of quantitative assessment makes them useless for distance comparisons. The same is true for the Sinclair experiments (34, 47).

Methods of evaluating free stimulus material did not reach

maturity until the 1940's (5, 6, 7, 8, 51, 52). The first distance experiment of sophisticated design (drawings as stimuli) was published by W. Carington in 1940 (5) and further similar research in subsequent papers (6, 7, 8, 10). The statistical assessments yielded several highly significant measures. The distances between target material and subjects varied widely from rather small to transatlantic.

Carington did not seem to be particularly interested in the distance factor and evaluated it only in a small, highly-selected portion of his data, Duke Group, Series IV-A (5). The twelve Duke subjects (longer distance) scored significantly better than the rest of the 105 participants, located in England and Holland. This result certainly falls in line with the *Zeitgeist*, but it does not reflect on the experiment as a whole because we do not know what distance trends may be hidden in the data. Moreover, Carington's methods, particularly his catalogue technique, have been severely criticized by S. G. Soal and F. Bateman (50) and by the Board of Review of the *Journal of Parapsychology* (4). In the face of these difficulties, I refrain from stating my personal position and present Carington's own conclusion: "Distance makes no difference" (9, p. 69).

Carington's experiments were repeated twice by A.S.P.R. experimenters. The first repetition, by E. Tayes, G. Murphy, and L. A. Dale (53), a large-scale experiment with 272 subjects and 8723 responses, did not yield significant results on the main measures taken. No distance breakdown was performed.

G. R. Schmeidler and L. W. Allison (44) undertook a second repetition in which they followed Carington's method more closely. His catalogue method, which has not withstood later criticism (50), was used. The experiment as a whole gave significant results ($P = .012$). When the agents and subjects were in the same building, the average score was .484; when the subjects were in "the New York area," the average score was .125; and when the subjects were scattered over the U.S.A. (including some in New York), the average score dropped to .062.

J. H. Rush and A. Jensen (42) performed a reciprocal drawing experiment of fifty trials over distances of 200-500 miles. Assessment gave $P = .002$. No distance comparisons were made.

M. C. Marsh developed a well-designed experiment with drawings as stimuli, and used a distance of 470 miles. This is documented in his Ph.D. dissertation (22) and also in a brief published report (23). Marsh's 371 subjects achieved "133 raw A hits on the targets used in the experiments, as against 37 hits on the unused control targets" (23, p. 8). Marsh used several statistical assessments which proved to be highly significant. No distance comparisons are possible.

On the whole, in distance experiments with free stimulus material the quality of the evaluation methods is very uneven; in many of them evaluation is inadequate by modern standards or lacking altogether. Therefore, this area of research is not a particularly fruitful one for probing the distance problem, although it does seem to give us some clues.

Trends in the Data Not Included in the Main Evaluations

The experiments not included in the main evaluations (Tables 1 and 2), but which have been briefly described, are so heterogeneous that unified quantitative analyses seem impossible. For that reason this section was nicknamed the "Tidbit Basket!" However, the experiments in which two distances were used lend themselves to comparison by a ranking method; that is, it is possible to determine which score at which distance within each experiment ranks higher. Table 3 summarizes the ranking efforts. Of the

Table 3
EXPERIMENTS USING TWO DISTANCES, NOT INCLUDED IN MAIN EVALUATIONS

Experiment	No. of Alternatives	Higher Score		Qualifying Comments
		Near	Far	
Shulman (45)	5	x		Sound stimuli for far targets, visual for near
Usher, Burt (56)	52	x		96 trials only
Nash (30)	10	x		Negative deviation numerically larger for far subjects
Michie, West (21)	12	x		Comparisons between successful (Fisk) and unsuccessful (West) experimenters
Bleakley (3)	various methods	x		Personal contact with subject only after far series
Carrington (5)	free material		x	Slanted selection of data; only one group (Duke) in one series evaluated for distance
Schmeidler, Allison (44)	free material	x		Use of Carrington catalogue method

seven experiments included in this table, five yielded higher scores on the near-distance targets, and two gave higher scores on the long-distance targets. Difficulties of interpretation are summarized in qualifying statements in the right-hand column of the table.

DISCUSSION

In a careful analysis of the relevant experimental data it was noted that ESP scores seem to decrease quite uniformly with increased distances. On the other hand, in sampling opinions about the relationship between ESP and distance, we find that most researchers express a view opposite to the direction the data take. W. Carrington's statement is quite representative of published opinion: "Distance makes no difference. All workers seem to be agreed on this" (9, p. 69). A few leading experimenters, however, such as S. G. Soal (50) and D. J. West (59), insert cautious clauses to the effect that it is likely, but not definitely established, that distance makes no difference. In the French school there have been dissident voices, among them those of R. Kherumian (19) and R. Amadou (1), who said: "Our conclusion would, then, be simple and prudent. It has not been proven whether distance affects or does not affect the intensity or the intelligibility of a telepathic message" (1, p. 223).

What is the reason for this discrepancy between the trends in the data and the majority of opinions? In part, as I pointed out above, it seems to be caused by our cultural inheritance, or *Zeitgeist*. Philosophers such as Kant, Schopenhauer, von Hartmann, and others have stressed the idea that ESP goes beyond the subjective forms which we call space and time. In part, the confusion seems to arise from our expectation that a decrease in ESP scores is an either/or situation: that a decrease in scores over distances either will be tremendous or will not occur at all. The logical possibility that a decrease in ESP efficiency could, in fact, be quite moderate has usually been overlooked. G. R. Schmeidler clearly perceived the difficulty of detecting such a decrease: "If, for example, psi is really proportional to distance or time raised to some fractional power, where the value of the fraction is small, very wide experimental variations will be necessary to detect any effect" (43, p. 111).

Another possible root of confusion may lie in unsystematic interpretations of the data. A logical way to deal with the results would be to ascertain carefully the trends in the data when distance between the stimulus and the percipient is introduced, and only thereafter to look into the causative factors responsible for the trends. For example, if the scores do decline with distance, we cannot tell immediately whether to ascribe this decline to subjective factors, such as attitudes, beliefs, feeling "on the spot," etc., or to the channel factor—the hypothetical energy which connects stimuli and percipient. In practice, we tend to jump quickly to this

second level of interpretation without carefully and thoroughly investigating at the first level. Very little work, if any, has been done on the third level of interpretation; that is, evaluating jointly the subjective factors and the channel factor, and their interactions; nor has much effort been made to analyze in a reliable way either the components of the subjective factors or the possible components of the channel factor. Some quite sharp theoretical analysis of the channel factor has been done, such as that of H. A. C. Dobbs (12), B. Hoffmann (17), J. H. Rush (41), and L. L. Vasiliev (57). Their hypotheses, however, have not been followed up experimentally, except in an often sporadic way by Vasiliev.

The need for clearly restating the problem and devising sharply delineated hypotheses and modern means for testing them is great indeed. Turner's model (see Appendix), upon which the main evaluations in this paper are based, is one attempt. Let us remember that Turner's approach keeps strictly within the data description level; that is, the first order of interpretation. So far as I know, he is the first to propose such a detailed and mathematically sound description of data concerning ESP over distances. He has carefully avoided any confounding of the first-order interpretation by the other higher-order interpretations mentioned above. I think this is a desirable approach.

It is possible to design experiments in which variables are ascertained that will permit mathematically precise data descriptions and also will generate second- or third-order interpretations. For example, in my forthcoming paper, to be published in the next issue of this *Journal*, I describe an experiment incorporating statistical control not only of some of the attitudes and personality variables, but also of some of the components of the channel variables, i.e., distance in space and time and their interactions. This is just a small beginning. Nevertheless, the twin problems of ESP over distance in space and time and of ESP orientation (how to find a particular point in space) present themselves as very challenging and promising tasks. A sound and systematic approach to these problems will probably establish clear-cut trends in the ESP channel factor. This would permit a definitive and precise theoretical analysis in scientific terms, which is not possible at present. Only when enough bricks are properly made can one build a sound structure.

There are breath-taking possibilities for a future theory of the ESP channel factor. For example, will an unknown energy be pinpointed with the aid of modern communication-theory tools such as precise energy expansion curves, signal-to-noise ratios, etc.? Will a new additive dimension be found in line with the hypotheses of Chari, Hart, and Smythies? Or will a qualitatively different

structure-giving principle emerge such as Driesch's *Seelenfeld*, Jung's "other-valued reality," or Tyrrell's "Elsewhere"? There is plenty of room for a variety of approaches to the channel factor. It seems to be a "many splendored thing."

This survey does not pretend to give final answers to questions concerning ESP and distance, or to the specific question of the relationship between distance and the channel factor in ESP. But it has succeeded, I hope, in its intent to clear the ground and establish the available empirical trends as a basis for new creative thinking and better future experiments.

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Appendix: A Statistical Model for Examining the Relations between ESP and Distance

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We consider a standard card-guessing experiment performed at several distances between objects being guessed and guessing subject. We suppose that it is of interest to investigate what relation (if any) holds between success at guessing and distance. We assume that the experiment has been carried out in such a way (the usual practice) that in the case of no ESP the chance of success on a single trial, p_c , is a constant at all distances (with the standard Duke cards $p_c = 0.2$), and the number of successes y_i out of n_i trials, at the i^{th} distance, has a binomial distribution.

Now, if ESP occurs, y_i will still have a binomial distribution (we will assume) but the probability of success will be changed from p_c to $p(x_i)$, a function of the distance x_i . For a binomial variate the mean is given by np . Thus, in the case of ESP we may say that the mean scoring level is changed from the chance level, $M_c(y_i) = n_i p_c$, to a new level, $M_x(y_i) = n_i p(x_i)$.

We now describe this change in mean scoring level in terms of the change, η , in probability entropy. We define (cf. 1, pp. 2-4) η by the relation

$$\eta_i = -\log_e [p(x_i)/p_c] \quad (1)$$

or, in terms of mean scoring level,

$$\eta_i = -\log_e [M_x(y_i)/n_i p_c] \quad (2)$$

We note that increasing the scoring level decreases the amount of probability entropy (the negative of the change in probability entropy is frequently referred to as the amount of information transmitted and, like the probability entropy itself, is measured in units determined by the base of the logarithm used in the definition (1) or (2); here the units are *natural units* or *nits*). Solving (2) for the mean scoring level at the i^{th} distance gives

$$M_x(y_i) = n_i p_c e^{-\eta_i} \quad (3)$$

By equations (2) and (3) we can express the change in probability entropy in terms of the mean scoring level, or the mean scoring level in terms of the change in probability entropy, at the i^{th} distance.

Let us now postulate a class of possible relations between the change in negative entropy and the distance, and hence by (3) between the mean scoring level and the distance. We suppose

that the change in negative entropy obeys the "general distance law":

$$\frac{d\eta_i}{dx_i} = \delta \frac{\eta_i}{x_i - \gamma\delta} \quad (4)$$

where δ is a proportionality constant and $\gamma\delta$ is an origin adjustment for the distance scale. The integrated form of (4) is found to be

$$\eta_i = -\pi(x_i - \gamma\delta)^\delta; \quad (5)$$

π is a constant we may term the "extrasensory perceptivity" and is a measure of the "power" of the object-subject system in a particular situation. It may be seen that the proportionality constant δ indexes the class of distance laws. For example, if $\delta = -2$, (5) becomes the familiar "inverse square law" so useful in describing the decrease in the radiant energy of light. If $\delta = 0$, no increase in probability entropy occurs (this case corresponds to the statement: "ESP is not related to distance"). If $\delta = -1$, the relation is hyperbolic; if $\delta = +1$, the relation is linear; if $\delta = +2$, the relation is parabolic; and if $|\delta| \rightarrow \infty$, the relation is exponential. All intermediate cases are also possible. A discussion of this family of curves has been given by Turner, Monroe, and Lucas (2).

Combining (3) and (5) we obtain the general distance law for mean scoring level

$$M_x(y_i) = n_i p_e e^{\pi(x_i - \gamma\delta)^\delta} \quad (6)$$

Consider two examples. If $\delta = 0$ (no effect of distance), (6) becomes

$$M_x(y_i) = n_i p_e e^\pi; \quad (7)$$

that is, the mean scoring level is equal to the mean chance scoring level multiplied by a factor which is the antilog of the extrasensory perceptivity. If $\delta = -1$ (the hyperbolic case), (6) becomes

$$M_x(y_i) = n_i p_e e^{\pi/(x_i + \gamma)}. \quad (8)$$

Other special cases follow similarly.

Now the practical problem of investigating the ESP-distance relation given a set of ESP scores at different distances can be solved using the model (6) and appropriate statistical procedures. Model (6) can be fitted to the data yielding estimates of the index of law δ (thus identifying the most plausible law). In addition confidence limits for δ can be obtained identifying the class of plausible laws. If these limits contain zero the hypothesis of no ESP-distance relation cannot be rejected. Estimates and confidence limits for the extrasensory perceptivity π will also be of interest.

Consider N scores $y_1, y_2, \dots, y_N$ at N distances $x_1, x_2, \dots, x_N$. The likelihood function is given by

$$\prod_{i=1}^N \binom{n_i}{y_i} \{p_e e^{\pi(x_i - \gamma\delta)^\delta}\}^{y_i} \{1 - p_e e^{\pi(x_i - \gamma\delta)^\delta}\}^{n_i - y_i} \quad (9)$$

That choice of values for γ , δ , and π which makes the likelihood function a maximum may be taken as the most plausible choice (likelihood principle). In practice finding such a choice may involve rather tedious computations.

A simpler approach will be developed here as an approximation to the use of the likelihood principle. We will restrict attention to the case $\gamma = 0$ (no distance adjustment needed; exponential law excluded) although this approach can be modified for the case γ nonzero.

We divide (6) through by $n_i p_e$, take logarithms twice and then set $\gamma = 0$. We obtain

$$\log_e \log_e \left(\frac{M_x(y_i)}{n_i p_e} \right) = \log_e \pi + \delta \log x_i \quad (10)$$

a linear form.

In (10) we now replace $M_x(y_i)$ by the observed estimate y_i , the scoring level at the i th distance. The relation (10) is no longer exact so we absorb the discrepancy by a residual element ϵ_i to give

$$\log_e \log_e \left(\frac{y_i}{n_i p_e} \right) = \log_e \pi + \delta \log x_i + \epsilon_i \quad (11)$$

a linear regression model. Note that (11) is defined only for $y_i > n_i p_e$. For π sufficiently large and for large samples this will usually be the case. For other cases the procedure will have to be modified.

Let the left side of (11) be denoted by Y_i and $\log x_i$ by X_i . Then we rewrite (11)

$$Y_i = \log_e \pi + \delta X_i + \epsilon_i \quad (12)$$

Now we will use standard linear regression procedures to fit (12), providing estimates of the slope δ and the intercept $\log_e \pi$ (and hence π itself), and to provide confidence limits for δ .

The variance of ϵ_i can be approximated using the following "propagation of error" argument

$$d\epsilon_i = dY_i = \frac{dy_i}{y_i \log(y_i/n_i p_e)};$$

and

$$V(\epsilon_i) = \frac{V(y_i)}{\{M_x(y_i) \log_e [M_x(y_i)/n_i p_e]\}^2}.$$

But the $V(y_i) = M_x(y_i)[1 - M_x(y_i)/n_i]$. So

$$V(\epsilon_i) = \frac{M_x(y_i)/n_i}{\{M_x(y_i) \log_e [M_x(y_i)/n_i p_e]\}^2} \quad (13)$$

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Since $V(\epsilon_i)$ is not constant it is appropriate to perform a weighted regression analysis using as weights $1/V(\epsilon_i)$. However, $V(\epsilon_i)$ is a function of the unknown constants π and δ ; so as a further approximation we suggest using as weights

$$w_i = \frac{y_i [\log_e(y_i/n_i p_0)]^2}{1 - y_i/n_i} \quad (14)$$

Now we compute in stepwise fashion:

(1) Raw sums

$$\begin{aligned} \sum w_i & \quad \sum w_i X_i & \quad \sum w_i Y_i \\ \sum w_i X_i^2 & \quad \sum w_i X_i Y_i & \quad \sum w_i Y_i^2 \end{aligned}$$

(2) Corrected sums

$$\begin{aligned} S_{xx} &= \sum w_i X_i^2 - \frac{(\sum w_i X_i)^2}{\sum w_i} \\ S_{xy} &= \sum w_i X_i Y_i - \frac{(\sum w_i X_i)(\sum w_i Y_i)}{\sum w_i} \\ S_{yy} &= \sum w_i Y_i^2 - \frac{(\sum w_i Y_i)^2}{\sum w_i} \end{aligned}$$

(3) Estimates of parameters

$$\begin{aligned} \delta &= S_{xy}/S_{xx} \\ \hat{\pi} &= \exp\{[\sum w_i Y_i - \delta \sum w_i X_i] / \sum w_i\} \end{aligned}$$

(4) Confidence limits for δ

$$\begin{aligned} SSR &= \delta S_{xy} \\ SSE &= S_{yy} - SSR \\ s_\delta^2 &= SSE / [(N-2)S_{xx}] \\ t_\alpha(N-2) &= t = \text{tabular value of "Student's" } t \text{ at the } \alpha \text{ significance level with } N-2 \text{ degrees of freedom} \end{aligned}$$

$$\delta - ts_\delta \leq \delta \leq \delta + ts_\delta \text{ with } 1 - \alpha \text{ confidence}$$

Then any value of δ lying within the limits of the above inequality has plausibility as great as or greater than $1 - \alpha$.

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